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Semi-Annual Status Report on Research Progress
NASA Research Grant NsG-2-59
April 1, 1965 to September 30, 1965

During the period covered by this status report, the following faculty members have derived support from NASA Research Grant NsG-2-59:

1. Professor Frederick J. Beutler received full-time support for two months in the summer, and 20% support during September. Earlier in the period, he received no grant support, as he was on sabbatical leave from The University of Michigan, visiting Stanford University, and the University of California at Berkeley.
2. Professor Lawrence L. Rauch was supported on a 25% basis during the months of April, July, and August, and on a 15% basis in September. He received no support in May or June.
3. Professor Root was supported on a nearly full-time basis during June and August, and approximately a quarter-time basis during April, May, and September.

Support was also furnished to Mr. Stuart Schwartz on a half-time basis; Mr. Schwartz has been working on his doctoral dissertation research under the supervision of Professor Root.

In April, Professor Beutler lectured at the University of California, Berkeley, and at the University of Southern California on the applications of stationary point processes to sampling and modulation of random signals; the subject matter of these lectures was based on the grant-supported research described in the status report covering October 1, 1964 to March 31, 1965. Several seminars on this topic, as well as on his earlier research on pseudo-inverses, were given at Stanford University. In each case, there was no cost to the grant.

Professor Beutler's research on stationary point processes (occurrence processes) has continued throughout the reporting period. This research has been carried on in collaboration with Dr. Oscar Leneman of Lincoln Laboratories. The theoretical work on axiomatic structure and mathematical properties of point processes is now essentially complete, and only awaits publication. The comprehensive theory will be presented in a lengthy paper that is now being revised for publication. A later paper will treat superposed processes, which consist of sums of such processes, and may have applications to multiplexing in modulation theory.

Current research on stationary point processes emphasizes applications to sampling and modulation processes. Recent calculations yield spectra for pulse-sampling

of random signals, where there are correlated errors in sampling amplitude, samples may be randomly omitted, and both read-in and read-out jitters are present in the sampling sequence. These computations should give more realistic results than those based on (the universally assumed) idealized sampling.

Other results along similar lines relate to minimum mean square realizable and non-realizable linear interpolators for the sampling and modulation processes mentioned in the preceding paragraph. Spectra and interpolators have also been derived for sampling with zero order hold (such as is sometimes used in control systems), where the sampling may again be subjected to jitter, skipping, and other random irregularities.

Theoretical results have been obtained on conditions under which error-free recovery of a signal can be obtained from samples. If the sampling times are precisely known, and there are no amplitude errors, error-free recovery is possible for many non-uniform sampling schemes, including those with jitter, skips, etc. For example, if the average sampling rate is even slightly above the Nyquist rate (even if sampling is irregular), the signal may be recovered from the samples during its past by a realizable operator. There are even some situations where realizable recovery is possible when the average rate is below the Nyquist rate.

During the reporting period, two papers of Professor Beutler appeared, representing the results of research accomplished with support from the grant. These were, "The Operator Theory of the Pseudo-Inverse I. Bounded Operators," Journal of Mathematical Analysis and Application, Vol. 10, No. 3, (1965) pp. 451-470, and "The Operator Theory of the Pseudo-Inverse II. Unbounded Operators with Arbitrary Range," Ibid. pp. 471-493. The requisite number of reprints of these papers have been furnished NASA.

Professor Rauch and his doctoral student, James K. Strozier, have made considerable progress in extending the previous work of Earl F. Smith on optimum demodulation of PCM. The mean absolute error and mean square error performance of the maximum inverse probability demodulator have been evaluated for the two three-bit word and two six-bit word case. The structures have been obtained for the minimum mean absolute error and minimum mean square error demodulators. Some of the error evaluation for these two demodulators has been obtained. The results are quite encouraging from the practical point of view in that the possibility is now established for suboptimal demodulators giving very considerable improvement over those in current use.

Professor Root presented an invited paper at the Symposium on System Theory, Polytechnic Institute of Brooklyn, in April. The work on this paper, entitled, "On System Measurement and Identification," was supported by the grant. The Proceedings of the Symposium are scheduled to appear shortly, and reprints of the paper will be forwarded as soon as they are available. The subject of this paper was the general problem of determining an unknown system from measurements of outputs when one is allowed to test the system by putting in known inputs subject to certain constraints. Both linear and nonlinear, and deterministic and stochastic systems were considered. The paper is partly expository, presenting known results of Wiener, Barrett, Bello and the author, organized around the question "What kind of prior information must be available about the unknown system to make its determination from the specified measurements possible?" This material includes a new combinatoric proof of certain properties

of general Hermite polynomials. In addition there are new results on the determination of nonlinear systems.

The abstract problems considered in the paper just mentioned were suggested to the author largely by the problem of determining appropriate characteristics of an unknown, time-varying radio communication channel, to be used in specifying data processing at the receiver. A paper on that subject entitled, "On the Measurement and Use of Time-Varying Communication Channels" by Professor Root appeared in Information and Control (v 8 no. 4, August 1965, pp 390-422). The work on this paper was partly supported by the grant. The material of this paper has been reported earlier, and copies have been forwarded to NASA.

A draft of a chapter entitled, "The Detection of Signals in Gaussian Noise," has been written with support from the grant by Professor Root, and copies have been forwarded to NASA. This chapter is scheduled to appear in a book being edited by Professor A.V. Balakrishnan of UCLA. It is a rigorous exposition of a part of what might be called the classical theory of signal detection. It does contain one previously unpublished result, due to the author, on the asymptotic behavior of matched filters.

Work on problems of communication through time-varying channels is being continued by Professor Root in collaboration with Dr. Reese Prosser of the M.I.T. Lincoln Laboratory.

Mr. Stuart Schwartz is working on a PhD dissertation under the direction of Professor Root on the subject of empirical Bayes decision theory applied to signal detection. This research is being supported by the grant. The general problem is to construct procedures and prove their validity for inferring an a priori distribution on a parameter while simultaneously performing a sequence of statistical decisions. It is one kind of unsupervised learning problem. The work has thus far used and extended results of H. Robbins* and others.

*See, e.g., Robbins, H., "The Empirical Bayes Approach to Statistical Decision Problems," Ann. Math. Stat. v 35, pp 1-20, 1964.